MATH 301 In-class activity: Absolute Value and ε neighborhoods

1. For each, say whether always true or could be possibly false. If false, give a counterexample and a possible corrected statement. If true, think of a proof.

(a) If $a < b$, then since $2 < 3$, we must have $a - 2 < b - 3$

(b) If $a < b$, then since $2 < 3$, we must have $a + 2 < b + 3$

(c) If $a < b$, then since $2 < 3$, we must have $2a < 3b$

(d) If $a < b$, then since $2 < 3$, we must have $a \div 2 < b \div 3$

2. For each, say whether always true or could be possibly false. If false, give a counterexample and a possible corrected statement. If true, think of a proof.

(a) If $|a| < x$ and $|b| < y$ then $|a - b| < x - y$

(b) If $|a + b| < x$ then $|a| + |b| < x$

(c) If $a + b < x$ then $|a + b| < x$

(d) If $|a| < x$ and $|b| < y$ then $|a| - |b| < x - y$

(e) If $|a - x| < 2$ and $|b - x| < 3$ then $|a + b| < 5$

(f) If $a + b < x$ and $-(a + b) < x$ then $(a + b)^2 < x^2$

3. For each, say whether always true or could be possibly false. If false, give a counterexample and a possible corrected statement. If true, think of a proof.

(a) Suppose $a \epsilon V_\varepsilon(b)$. Then $b \epsilon V_\varepsilon(a)$.

(b) Suppose $a \epsilon V_\varepsilon(b)$. Then $b \epsilon V_\delta(a)$ for any $\delta \leq \varepsilon$.

(c) Suppose $x \epsilon V_\varepsilon(a) \cap V_\varepsilon(b)$. Then x lies between a and b.

(d) Suppose $x \epsilon V_\varepsilon(a) \cup V_\varepsilon(b)$. Then $\max\{|x - a|, |x - b|\} \leq \varepsilon$.

(e) Suppose $V_\varepsilon(a) \subset V_\delta(b)$. Then $\varepsilon \leq \delta$.

(f) Suppose $V_\varepsilon(a) \subset V_\delta(b)$. Then $|a - b| < \delta$.